Practice Midterm Solutions

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1 Problem 1

(a) Plugging in the parameterization of the line into the equation of the plane we get

$$2 \cdot (1 + 3 \cdot t) - 3 \cdot (t + 2) + (2t - 4) = 5$$

collecting terms

$$5t - 8 = 5$$

thus

$$t = \frac{13}{5}.$$

Plugging back in to the parameterization of the line the intersection is the point P

$$P = (1 + 3 \cdot \frac{13}{5}, 2 + \frac{13}{5}, 2 \cdot \frac{13}{5} - 4)$$

thus

$$P = (\frac{44}{5}, \frac{23}{5}, \frac{6}{5}).$$

(b) A normal vector of the plane is the vector $\vec{v} = (2, -3, 1)$, if $\vec{m} = (3, 1, 2)$ then the vector

$$\vec{m}_{||,\vec{v}} = \frac{\vec{v} \cdot \vec{m}}{\vec{v} \cdot \vec{v}} \cdot \vec{v} = \frac{5}{14} \cdot (2, -3, 1) = (\frac{10}{14}, -\frac{15}{14}, \frac{5}{14}).$$

So the projection onto the plane is (by definition!) the vector $\vec{m}_{\perp,\vec{v}}$ and we can calculate

$$\vec{m}_{\perp,\vec{v}} = \vec{m} - \vec{m}_{||,\vec{v}} = (\frac{32}{14}, \frac{29}{14}, \frac{23}{14}).$$

2 Problem 2

The answer is $\frac{\sqrt{3}}{2}$. One can take the cross product of

$$(\hat{j} - \hat{i}) \times (\hat{k} - \hat{i}) = \hat{i} + \hat{k} + \hat{j} = (1, 1, 1)$$

this vector has magnitude $\sqrt{3}$, thus the area of the triangle (half the area of the corresponding parallelogram) is $\frac{\sqrt{3}}{2}$.

3 Problem 3

(a) Explicitly calculating this cross product we get

$$\vec{v}'(t) = (\hat{j} + \vec{v}(t) \times \hat{k})$$

expanding this we see that

$$\vec{v}(t) = (v_x(t), v_y(t), v_z(t))$$

where

$$v_z'(t) = 0$$
$$v_x'(t) = v_y(t)$$
$$v_y'(t) = 1 - v_x(t).$$

Solving the first equation we see that $v_z(t) = \hat{k} \cdot \vec{v}(t) = 0$ for any time t.

(b) We calculate the derivative of the square of the norm

$$\frac{d}{dt}(\vec{w}(t) \cdot \vec{w}(t)) = 2\vec{w}(t) \cdot \vec{w}\,'(t) = 2(v_x(t) - 1) \cdot v_x'(t) + 2 \cdot v_y(t) \cdot v_y'(t)$$

plugging in the equations from part (a) and simplifying we see that this quantity is zero.

(c) Given $\vec{v}(t) = (1 + (v_0 - 1)\cos(t), -(v_0 - 1)\sin(t), 0)$ we just apply the fundamental theorem of calculus as in class:

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(t)\, dt.$$

Integrating we get

$$\vec{r}(t) = (t + (v_0 - 1)\sin(t), (v_0 - 1)(\cos(t) - 1), 0)$$